Section 11.2: Arithmetic Sequences and Series

Video 1: Arithmetic Sequences

An **arithmetic sequence** is a sequence in which each term differs from the preceding term by a fixed constant (**common difference, *d***).

$$\underset{a_1}{3}, \overset{+4}{7}, \overset{+4}{11}, 15, \ldots \qquad d = 4$$

The common difference of an arithmetic sequence is given by $d = a_n - a_{n-1}$.

1) List the first 5 terms of the given arithmetic sequence.

a) The first term is 8 and the common difference is 10.

8, 18, 28, 38, 48

b) $a_1 = 6$, $d = -5$

6, 1, -4, -9, -14

Video 2: nth Term of an Arithmetic Sequence

In an arithmetic sequence with first term a_1 and common difference *d*, the *n*th term a_n is given by the following:

$$\underline{a_n} = a_1 + \underline{(n-1)} \cdot d$$

2nd 1 d
3rd 2 d
4th 3 d

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots$

2) Find a_{19} and a_n for the given arithmetic sequence: $-9, -4, 1, 6, \ldots$

a_1

$-4-(-9)$
$1-(-4)$
$6-1 = 5 = d$

$a_{19} = a_1 + 18 \cdot d$
$= -9 + 18 \cdot 5$
$= -9 + 90$
$= 81$

$a_n = a_1 + (n-1) \cdot d$
$a_n = -9 + (n-1) \cdot 5$

$\downarrow$

$a_{19} = -9 + (19-1) \cdot 5$
$= -9 + 18 \cdot 5 = 81$

3) If an arithmetic sequence has terms $a_2 = 16$ and $a_3 = 9$, find a_n and a_{12}.

$d = a_3 - a_2 = 9 - 16 = -7$

$a_1 + d = a_2$
$a_1 + (-7) = 16$
$\quad +7 \quad +7$
$a_1 = 23$

$a_n = a_1 + (n-1) \cdot d$
$a_n = 23 + (n-1)(-7)$

$a_{12} = 23 + (12-1)(-7)$
$= 23 + 11(-7)$
$= 23 - 77$
$= -54$

$a_n = 23 - 7n + 7$
$a_n = 30 - 7n$
$a_{12} = 30 - 7(12)$

$6\ d's$

4) If an arithmetic sequence has terms $a_9 = 22$ and $a_{15} = 76$, find a_1.

$$6 \cdot d = 76 - 22$$
$$6d = 54$$
$$d = 9$$

$$a_9 = a_1 + (9-1) \cdot d$$
$$22 = a_1 + 8 \cdot 9$$
$$22 = a_1 + 72$$
$$-72 \qquad -72$$
$$-50 = a_1$$

Video 3: Arithmetic Series

If an arithmetic sequence has first term a_1 and common difference *d*, the sum S_n of the first *n* terms is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

5) Find the sum of the first 100 positive integers.

$a_1 \rightarrow 1, 2, 3, \ldots, 100$

$d = 1$ $\qquad a_{100} = 100$

$$S_{100} = \frac{100}{2}(1 + 100)$$
$$= \frac{\overset{50}{\cancel{100}} \cdot 101}{\cancel{2}} = 5050$$

$1, 2, 3, \ldots, 98, 99, 100$

50 101's

6) Find the sum of the first 15 odd positive integers.

$a_1 = 1 \searrow 1, 3, 5, \ldots$

$d = 2$

$$a_n = a_1 + (n-1) \cdot d$$
$$= 1 + (n-1) \cdot 2$$
$$a_{15} = 1 + (15-1) \cdot 2$$
$$= 1 + 14 \cdot 2$$
$$= 29$$

$$S_{15} = \frac{15}{2}(1 + 29)$$
$$= \frac{15 \cdot \overset{15}{\cancel{30}}}{\underset{1}{\cancel{2}}} = 225$$

7) Find S_{15} for the arithmetic sequence $\underbrace{\overbrace{-13,-5}^{8},3}_{8}\overbrace{,11}^{8},\ldots$

$a_1 = -13$

$d = 8$

$a_n = a_1 + (n-1)\cdot d$

$a_{15} = -13 + (15-1)\cdot 8$

$= -13 + 14\cdot 8$

$= -13 + 112$

$= 99$

$S_{15} = \frac{15}{2}(a_1 + a_{15})$

$= \frac{15}{2}(-13+99)$

$= \frac{15\cdot \cancel{86}^{43}}{\cancel{2}_{1}}$

$= 645$

8) If the sum of the first 20 terms of an arithmetic sequence is 3600 and $a_{20}=351$, find a_1 and d.

$S_{20} = 3600$

$a_{20} = 351$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{20} = \frac{20}{2}(a_1 + 351)$

$3600 = 10(a_1 + 351)$

$360 = a_1 + 351$

$9 = a_1$

$a_n = a_1 + (n-1)d$

$n: 20 \quad a_{20} = a_1 + (20-1)d$

$351 = 9 + 19d$

$\underline{-9 \quad -9}$

$\frac{342}{19} = \frac{19d}{19}$

$d = 18$

Video 4: Summation Notation and Arithmetic Series

9) Evaluate each sum.

a) $\sum_{i=1}^{14}(6i+11)$

$n=14$

$d=6$

$a_1 = 6(1)+11$
$= 17$

$a_{14} = a_1 + (14-1)d$
$a_{14} = 17 + 13 \cdot 6$
$a_{14} = 17 + 78$
$a_{14} = 95$

$S_{14} = \frac{14}{2}(a_1 + a_{14})$
$S_{14} = \frac{\cancel{14}^{7}}{\cancel{2}}(17+95)$
$S_{14} = 7(112)$
$= 784$

b) $\sum_{i=5}^{10}(7-2i)$

$d=-2$

$i=5 \quad 7-2(5) = -3 \leftarrow a_1$

$i=5 \rightarrow i=10$
6 terms

Need S_6
$a_1 = -3 \quad d=-2$
$a_6 = ?$

$a_6 = a_1 + (6-1)d$
$= -3 + 5(-2)$
$= -13$

$S_6 = \frac{6}{2}(-3 + (-13))$
$= 3(-16)$
$= -48$